### A Comparison of Two Guidance Strategies for Autonomous Vehicles

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# MOTIVATION

- 90% of the road accidents are due to the human errors.
- Road safety should be improved.
- Autonomous vehicles are considered as promising way for the Intelligent transportation systems.
- Lateral dynamic control should be improved.











# OUTLINE

- 1. Dynamic vehicle modeling
- 2. Errors model
- 3. Control design
- 4. Simulation tests
- 5. Conclusion & Outlooks

### DYNAMIC VEHICLE MODELING

Linear bicycle model is used for the controller synthesis.

![](_page_7_Picture_6.jpeg)

![](_page_7_Figure_7.jpeg)

Lateral vehicle dynamics

$$\sum M = L_f F_f - L_r F_r$$

# DYNAMIC VEHICLE MODELING

**Bicycle model** 

![](_page_8_Figure_7.jpeg)

#### Input control

 $\partial_f$  Front steering wheel-angle.

#### Output vector

- $v_y$  Lateral velocity.
- $\psi$ Yaw rate.

#### Remark

 $v_{r}$  Longitudinal velocity is constant.

#### **CoP** position

Its position depends on the vehicle parameters

$$x_{cop} = \frac{I_z}{L_f m}$$

![](_page_9_Figure_8.jpeg)

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Its position depends on the vehicle parameters 

 $x_{cop} = \frac{I_z}{L_f m}$ 

#### The rear tire lateral force has two effects on the system dynamics

- $a_y$  lateral acceleration along the body of the vehicle.
- $\ddot{\psi}$  angular acceleration around the vehicle's CoG.

![](_page_10_Figure_12.jpeg)

### **CoP** position

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### The rear tire lateral force has two effects on the system dynamics

- $a_y$  lateral acceleration along the body of the vehicle.
- $\ddot{\psi}$  angular acceleration around the vehicle's CoG.

At the CoP, these two effects cancel each other out.  $a_y^r - x_{cop}\ddot{\psi}^r = 0$ 

![](_page_11_Figure_13.jpeg)

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### The rear tire lateral force has two effects on the system dynamics

- $a_y$  lateral acceleration along the body of the vehicle.
- $\ddot{\psi}$  angular acceleration around the vehicle's CoG.

![](_page_12_Picture_12.jpeg)

![](_page_12_Figure_13.jpeg)

#### **Benefits**

- Using the CoP allows to preview the lateral error (look ahead).
- Using the CoP does not require the knowledge of the rear tire lateral force.

- References
  - $\psi_{ref}$  desired yaw angle.  $\dot{\psi}_{ref}$  desired yaw rate.
- Orientation error  $e_{\psi} = \psi - \psi_{ref}$
- CoG lateral error dynamic

 $\dot{e}_y = v_y + v_x e_\psi$ 

CoP lateral error dynamic

$$\dot{e}_{cop} = \dot{e}_y + x_{cop} \dot{e}_\psi$$

![](_page_13_Figure_11.jpeg)

![](_page_14_Figure_4.jpeg)

![](_page_15_Figure_4.jpeg)

![](_page_16_Figure_4.jpeg)

#### Remark

The contribution of the control input will be more important in the CoP case than in the CoG case due to the  $R_l > 1$  term.

![](_page_17_Figure_4.jpeg)

#### Remark

 $\psi_{ref}$  acts on the error model as a disturbance.

![](_page_18_Figure_4.jpeg)

# SIMULATION TEST ON OPEN LOOP

#### Objective

 Compare the behavior of CoGM and CoPM in a lane departure situation.

#### **Simulation Conditions**

- Lane departure situation.
- Reference trajectory is straight line.
- Constant speed 15 m/s.
- Constant steering wheel angle 5 deg.

![](_page_19_Figure_12.jpeg)

# SIMULATION TEST ON OPEN LOOP

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 Compare the behavior of CoGM and CoPM in a lane departure situation.

#### **Simulation Conditions**

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#### Results

- The lateral error at the CoP is bigger than the lateral error at the CoG.
- The orientation errors are the same in both models.

![](_page_20_Figure_15.jpeg)

# **CONTROL DESIGN**

$$\frac{d}{dt}\xi(t) = A\xi(t) + B\delta_f(t) + \begin{bmatrix} 0\\d_2\\0\\d_4 \end{bmatrix} \dot{\psi}_{ref}$$

- ψ<sub>ref</sub> desired yaw angle.
  ψ<sub>ref</sub> desired yaw rate.

#### Proposed control law

 $\delta_f(t) = u_{FF}(t) + u_{FB}(t)$ 

- Feed-Forward aims to eliminate the effect of the disturbance on a part of the state vector.
- Robust State-Feedback aims to stabilize the system in closed loop and to attenuate the effect of the disturbance.

![](_page_21_Figure_13.jpeg)

### **CONTROL DESIGN: FEED-FORWARD**

• CoG Model  

$$u_{FF}(t) = \frac{m}{C_f} \left( \frac{C_f L_f - C_r L_r}{m v_x} + v_x \right) \dot{\psi}_{ref}(t)$$
• CoP Model  

$$u_{FF}(t) = \frac{m}{C_f} \left( R_l \frac{C_f L_f}{m v_x} + v_x \right) \dot{\psi}_{ref}(t)$$

## **CONTROL DESIGN: FEED-FORWARD**

- CoG Model  

$$u_{FF}(t) = \frac{m}{C_f} \left( \frac{C_f L_f - C_r L_r}{m v_x} + v_x \right) \dot{\psi}_{ref}(t) \qquad - CoP \text{ Model}$$

$$u_{FF}(t) = \frac{m}{C_f} \left( R_l \frac{C_f L_f}{m v_x} + v_x \right) \dot{\psi}_{ref}(t)$$

#### By applying the control law

$$\begin{split} \delta_{f}(t) &= u_{FF}(t) + u_{FB}(t) \\ \frac{d}{dt}\xi(t) &= A\xi(t) + Bu_{FB}(t) + Bu_{FF}(t) + \begin{bmatrix} 0 \\ d_{2} \\ 0 \\ d_{4} \end{bmatrix} \dot{\psi}_{ref} \quad \longrightarrow \quad \frac{d}{dt}\xi(t) &= A\xi(t) + Bu_{FB}(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ d_{4} \end{bmatrix} \dot{\psi}_{ref} \end{split}$$

# **CONTROL DESIGN: ROBUST STATE FEEDBACK**

$$\frac{d}{dt}\xi(t) = A\xi(t) + Bu_{FB}(t) + \begin{bmatrix} 0\\0\\0\\d'_4 \end{bmatrix} \dot{\psi}_{ref}$$

Robust state feedback action 
$$u_{FB}(t) = -K\xi(t)$$

#### Objective

- Guarantee a decay rate exponential convergence  $\alpha$  of the state vector  $\xi(t)$ .  $\exists \alpha > 0: \quad \dot{V}(t) + 2\alpha V(t) < 0$
- Guarantee an attenuation level  $\gamma$  of the disturbance  $\dot{\psi}_{ref}$  on the state  $\dot{e}_{\psi}$ .

 $||\dot{e}_{\psi}||_{2}^{2} < \gamma^{2} \left| \left| \dot{\psi}_{ref} \right| \right|_{2}^{2}$ 

Lyapunov candidate  

$$V(t) = \xi^T(t) P \xi(t)^{\text{and}}$$
  
 $P = P^T > 0$ .

# **CONTROL DESIGN: ROBUST STATE FEEDBACK**

$$\frac{d}{dt}\xi(t) = A\xi(t) + Bu_{FB}(t) + \begin{bmatrix} 0\\0\\0\\d_4' \end{bmatrix} \dot{\psi}_{ref}$$

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$$||\dot{e}_{\psi}||_{2}^{2} < \gamma^{2} ||\dot{\psi}_{ref}||_{2}^{2}$$

#### **Problem formulation**

$$\begin{bmatrix} (A - BK)^T P + P(A - BK) + 2\alpha P + R^T R & PD' \\ (PD')^T & -\gamma I \end{bmatrix} < 0$$

Lyapunov candidate  $V(t) = \xi^T(t)P\xi(t)^{\text{and}}$  $P = P^T > 0$ .

#### Trade-off between

 $\alpha$  Large decay rate and  $\gamma < 1$ .

#### Double lane change maneuver

• The test consists in performing a double lane change maneuver at different speeds.

#### **Simulation Conditions**

- The track supposed to be flat.
- No vertical nor load transfer are considered.
- A 2D model is used for simulation purpose (with saturation on the lateral tire forces ).

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#### Robust state feedback design

- Controllers are designed at the constant speed  $v_x = 25m/s$  (nominal speed).
- The LMI problem is programmed thanks to the Yalmip interface (Lofberg, 2004) coupled with the SeDuMi solver (Sturm, 1999).
- Decay rate  $\alpha = 0.2$ .
- Attenuation level  $\gamma = 0.3$ .

![](_page_28_Figure_6.jpeg)

#### Simulation test at a different speed $v_x = 10m/s$ .

![](_page_29_Figure_7.jpeg)

![](_page_29_Figure_8.jpeg)

![](_page_29_Figure_9.jpeg)

![](_page_29_Picture_10.jpeg)

-Using CoG strategy Using CoP strategy

#### **Results**

- Both strategies are robust with respect to the longitudinal speed variation.
- The CoP strategy still offers an effective trajectory tracking in terms of the lateral error.

# **CONCLUSION & OUTLOOKS**

#### Conclusion

- The CoP strategy ensures a better trajectory tracking and anticipates the lateral position error.
- Both strategies are robust with respect to the longitudinal speed variation.

#### Future works

• Enhance the lateral stability in critical situation by using the CoP strategy.

### THANKS FOR YOUR ATTENTION ! QUESTION ?

![](_page_31_Picture_1.jpeg)