### **A Comparison of Two Guidance Strategies for Autonomous Vehicles**

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## MOTIVATION

- **90%** of the road accidents are due to the **human errors.**
- **Road safety** should be improved.
- **Autonomous vehicles** are considered as promising way for the Intelligent transportation systems.
- **Lateral dynamic control** should be improved.











## **OUTLINE**

- 1. Dynamic vehicle modeling
- 2. Errors model
- 3. Control design
- 4. Simulation tests
- 5. Conclusion & Outlooks

### DYNAMIC VEHICLE MODELING

Linear bicycle model is used for the controller synthesis.





**Lateral vehicle dynamics**

$$
\sum M = L_f F_f - L_r F_r
$$

## DYNAMIC VEHICLE MODELING

**Bicycle model**



#### **Input control**

 $\delta_f$  Front steering wheel-angle.

#### **Output vector**

- $v_y$  Lateral velocity.
- ψ Yaw rate.
- **Remark**
	- $v<sub>r</sub>$  Longitudinal velocity is constant.

### **CoP position**

**Its position depends on the vehicle parameters** 

$$
x_{cop} = \frac{I_z}{L_f m}
$$



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### **The rear tire lateral force has two effects on the system dynamics**

- allateral acceleration along the body of the vehicle.
- $\dot{\psi}$  angular acceleration around the vehicle's CoG.



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At the CoP, these two effects cancel each other out.

$$
y^r_y - x_{cop}\ddot{\psi}^r = 0
$$



### **Benefits**

- Using the CoP allows to preview the lateral error (look ahead) .
- **Using the CoP does not require the knowledge of** the rear tire lateral force.

- **References**
	- $\psi_{ref}$  desired yaw angle.  $\dot{\psi}_{ref}$  desired yaw rate.
- **Orientation error**  $e_{\psi} = \psi - \psi_{ref}$
- **CoG lateral error dynamic**

 $\dot{e}_y = v_y + v_x e_{\psi}$ 

**CoP lateral error dynamic**

$$
\dot{e}_{cop} = \dot{e}_y + x_{cop} \dot{e}_{\psi}
$$









#### **Remark**

The contribution of the control input will be more important in the CoP case than in the CoG case due to the  $R_l > 1$  term.



#### **Remark**

 $\dot{\psi}_{ref}$  acts on the error model as a disturbance.



## SIMULATION TEST ON OPEN LOOP

#### **Objective**

 Compare the behavior of CoGM and CoPM in a lane departure situation.

#### **Simulation Conditions**

- Lane departure situation.
- **Reference trajectory is straight line.**
- Constant speed  $15 m/s$ .
- **Constant steering wheel angle 5 deg.**



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#### **Results**

- **The lateral error at the CoP is bigger than** the lateral error at the CoG.
- **The orientation errors are the same in both** models.



# CONTROL DESIGN

$$
\frac{d}{dt}\xi(t) = A\xi(t) + B\delta_f(t) + \begin{bmatrix} 0\\d_2\\0\\d_4 \end{bmatrix} \dot{\psi}_{ref}
$$

**References**

- $\cdot$   $\;\psi_{ref}$  desired yaw angle.
- $\dot{p}_{ref}$  desired yaw rate.

#### **Proposed control law**

 $\delta_f(t) = u_{FF}(t) + u_{FB}(t)$ 

- Feed-Forward aims to eliminate the effect of the disturbance on a part of the state vector.
- Robust State-Feedback aims to stabilize the system in closed loop and to attenuate the effect of the disturbance.



### CONTROL DESIGN: FEED-FORWARD

• **CoG Model**  
\n
$$
u_{FF}(t) = \frac{m}{C_f} \left( \frac{C_f L_f - C_r L_r}{m v_x} + v_x \right) \dot{\psi}_{ref}(t)
$$
\n
$$
u_{FF}(t) = \frac{m}{C_f} \left( R_l \frac{C_f L_f}{m v_x} + v_x \right) \dot{\psi}_{ref}(t)
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\n• **CoP Model**  
\n
$$
u_{FF}(t) = \frac{m}{C_f} \left( R_l \frac{C_f L_f}{m v_x} + v_x \right) \dot{\psi}_{ref}(t)
$$

### **By applying the control law**

$$
\delta_f(t) = u_{FF}(t) + u_{FB}(t)
$$
\n
$$
\frac{d}{dt}\xi(t) = A\xi(t) + Bu_{FB}(t) + Bu_{FF}(t) + Bu_{FF}(t) + \begin{bmatrix} 0 \\ d_2 \\ 0 \\ d_4 \end{bmatrix} \dot{\psi}_{ref}
$$
\n
$$
\frac{d}{dt}\xi(t) = A\xi(t) + Bu_{FB}(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ d'_4 \end{bmatrix} \dot{\psi}_{ref}
$$

# CONTROL DESIGN: ROBUST STATE FEEDBACK

$$
\frac{d}{dt}\xi(t) = A\xi(t) + Bu_{FB}(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ d'_4 \end{bmatrix} \dot{\psi}_{ref}
$$

$$
\fbox{\textbf{Robust state feedback action}}\\ u_{FB}(t)=-K\xi(t)
$$

#### **Objective**

- Guarantee a decay rate exponential convergence  $\alpha$  of the state vector  $\xi(t)$ .  $\exists \alpha > 0: \quad \dot{V}(t) + 2\alpha V(t) < 0$
- Guarantee an attenuation level  $\gamma$  of the disturbance  $\dot{\psi}_{ref}$  on the state  $\dot{e}_{\psi}.$

 $||\dot{e}_{\psi}||_2^2 < \gamma^2 ||\dot{\psi}_{ref}||_2^2$ 

**Lyapunov candidate**  
\n
$$
V(t) = \xi^{T}(t) P \xi(t)
$$
\n
$$
P = P^{T} > 0
$$

## CONTROL DESIGN: ROBUST STATE FEEDBACK

$$
\frac{d}{dt}\xi(t) = A\xi(t) + Bu_{FB}(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ d'_4 \end{bmatrix} \dot{\psi}_{ref}
$$

$$
\boxed{\text{Robust state feedback action}}\\ u_{FB}(t) = -K\xi(t)
$$

#### **Objective**

- Guarantee a decay rate exponential convergence  $\alpha$  of the state vector  $\xi(t)$ .  $\exists \alpha > 0: \quad \dot{V}(t) + 2\alpha V(t) < 0$
- Guarantee an attenuation level  $\gamma$  of the disturbance  $\dot{\psi}_{ref}$  on the state  $\dot{e}_{\psi}.$

$$
\left|\left|\dot{e}_{\psi}\right|\right|_{2}^{2} < \gamma^{2} \left|\left|\dot{\psi}_{ref}\right|\right|_{2}^{2}
$$

#### **Problem formulation**

$$
\begin{bmatrix} (A - BK)^T P + P(A - BK) + 2\alpha P + R^T R & P D' \\ (P D')^T & -\gamma I \end{bmatrix} < 0
$$

**Lyapunov candidate**  $V(t) = \xi^T(t) P \xi(t)$  and  $P = P^T > 0$ .

#### **Trade-off between**

 $\alpha$  Large decay rate and  $\gamma < 1$ .

#### **Double lane change maneuver**

The test consists in performing a double lane change maneuver at different speeds.

#### **Simulation Conditions**

- **The track supposed to be flat.**
- No vertical nor load transfer are considered.
- A 2D model is used for simulation purpose (with saturation on the lateral tire forces ).

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#### **Robust state feedback design**

- Controllers are designed at the constant speed  $v_x = 25m/s$  (nominal speed).
- The LMI problem is programmed thanks to the Yalmip interface (Lofberg, 2004) coupled with the SeDuMi solver (Sturm, 1999).
- **Decay rate**  $\alpha = 0.2$ **.**
- Attenuation level  $\nu = 0.3$ .



### Simulation test at a <u>different speed  $v_x = 10m/s$ .</u>



 $-50$ 

 $-100$ <sub>0</sub>

20

40

60

 $X(m)$ 

80

**100** 

120





#### **Results**

- Both strategies are robust with respect to the longitudinal speed variation.
- **The CoP strategy still offers an effective trajectory** tracking in terms of the lateral error.

## CONCLUSION & OUTLOOKS

#### **Conclusion**

- The CoP strategy ensures a better trajectory tracking and anticipates the lateral position error.
- Both strategies are robust with respect to the longitudinal speed variation.

#### **Future works**

Enhance the lateral stability in critical situation by using the CoP strategy.

### THANKS FOR YOUR ATTENTION ! QUESTION ?

